Non-linear Moving Horizon State Estimation and Control for the Superfluid Helium Cryogenic Circuit at the Large Hadron Collider

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Abstract—We consider the problem of temperature stabilization of the Large Hadron Collider’s (LHC) superconducting magnets, cooled with superfluid helium 4 (He II). The temperature dynamics is highly non-linear, exhibiting inverse response, variable dead-times, strong couplings between process variables and strong saturation effects. Multiple operational constraints must be respected. We present a prototype application of a hybrid Non-linear Moving Horizon State Estimation - Luenberger observer (MHSE-LO) and Non-linear Model Predictive Control (NMPC). It is based on: 1) a detailed, low computing cost, first principles model of the temperature dynamics and 2) a simple optimization approach tailored for systems with stiff dynamics. The prototype control system has been tested at the LHC, demonstrating excellent, robust performance.

I. INTRODUCTION

The 27 km circumference Large Hadron Collider (LHC) is the most powerful particle accelerator and collider in the World. Performance of its main superconducting magnets has been boosted by cooling them with superfluid helium 4 (He II). He II exhibits extraordinary physical properties but it is available only below the phase transition temperature $T_{\lambda_{\text{II}}} = 2.16$ K [9]. Using 80 tons of He II, the Superfluid Helium Cryogenic Circuit (SHCC) stabilizes at 1.9 K the temperature of 37'000 tons of equipment [2].

Temperature dynamics of the long strings of magnets is highly non-linear, non-self-regulating and exhibits varying inverse response and dead-times, strong couplings between process variables and saturation effects. The non-linearity is pronounced because the operating point shifts: 1) due to perturbations present during stable operation and 2) during significant transient phases. Moreover, multiple operational constraints must be respected and instrumentation is installed sparsely. The PI controllers currently used to stabilize the temperature perform sufficiently well during stable operation, however their performance is poor during transients.

Moving Horizon State Estimation (MHSE) is a state estimation technique where trajectories of unknown states and perturbations are repeatedly optimized over a finite past time horizon using a non-linear model of system dynamics and available measurements [5]. Non-linear Model Predictive Control (NMPC) is a feedback control technique where trajectories of manipulated variables are repeatedly optimized over a finite future time horizon using a non-linear model of system dynamics, available measurements and estimates and reference trajectories for controlled variables [3]. These advanced control techniques directly address non-linear dynamics, complex interactions between process variables and operational constraints. The total computational complexity of the model simulation and numerical solvers used in the optimizations must allow for its real-time feasibility.

Motivated by variable dead-time and strong inverse response of the non-linear temperature dynamics at the 30 m long LHC prototypes, NMPC and MHSE have been developed using a first-principles, lumped parameter model of the dynamics, having 2 state variables, and an off-the-shelf optimization procedure [1]. Then, a distributed-parameters model of the temperature dynamics at a 214-m long sub-sector of the SHCC at the LHC, having 10 states, has been used in the first version of NMPC and MHSE unsuccessfully tested at the LHC, proving that real-time feasibility may be achieved with a more complex, distributed parameters model if a tailored optimization algorithm is applied [7].

This paper presents the first ever working prototypes of MHSE and NMPC applied to the temperature stabilization in an SHCC sub-sector at the LHC. It is based on: 1) a detailed, first-principles, thermo-hydraulic, distributed-parameters, numerical model of the temperature dynamics having 262 states, carefully implemented to minimize the computational cost of its simulation, 2) an unconventional hybrid MHSE-Luenberger Observer (MHSE-LO) approach and 3) tailored optimization algorithms. Tested at the LHC, the advanced control system exhibits excellent, robust performance far better than that of currently used PI controllers. After this introduction, Section II presents the cryogenic circuit. Then, Section III describes the model. The MHSE-LO and NMPC are presented in Section IV. Section V describes the experimental implementation and results. Finally, Section VI concludes the paper an gives an outlook.

II. THE SUPERFLUID HELIUM CRYOGENIC CIRCUIT

Design of the SHCC, also known as the 1.8 K Cooling Loop, is based on the novel concept of a bayonet heat exchanger, integrated into a static bath of pressurized He II together with the cooled superconducting magnet coils, see Figures 1 and 2 [4]. Supercritical helium at 4.5 K, 400 kPa is supplied from a refrigerator, through a 3 km long distribution line to 23, 106.9 m long standard cells of the circuit, distributed along a 2.5 km long arc of the LHC. Then, at each cell, this helium is sub-cooled, throttled at a control valve and the resulting two-phase helium is feed through an over 100 m long, 10 mm inner diameter pipe into a larger 54 mm inner diameter, over 100 m long tube of the bayonet
heat exchanger. Very low pressure of 1.6 kPa is maintained in the heat exchanger tube, forcing the He II flowing inside to boil at 1.8 K, taking away the heat transferred from the bath. The created vapor is sucked away through the phase separator, the sub-cooling heat exchanger and the distribution line back to the refrigerator. One bayonet heat exchanger per standard cell is installed, running through 6 cryo-dipoles and 2 Short-Straight-Sections housing 8 main superconducting magnets. The bath is shared by 2 or 3 cells composing a sub-sector. The maximal magnets temperature in each cell is regulated using a PI controller manipulating the valve.

Figure 3 shows an LHC sub-sector during stable operation with particle beam energy of 3.5 TeV, when strongest perturbations occur during ramping up and down of the magnets current. The 16 magnets temperatures are level while the very low pressure is degraded and lost. When the pressure measured at the refrigerator rises, the corresponding unmeasured helium saturation temperature in the heat exchanger increases. The set-point must be kept above the this saturation temperature in order to avoid coolant accumulation in the phase separator. The margin of approx. 0.1 K kept between the saturation temperature at the refrigerator and the set-point takes into account an unknown pressure drop in the distribution line. If the maximal temperature in an LHC arc exceeds 2.1 K, the magnets powering is not permitted. When it rises above $T_{sp}$, superfluidity is lost and pronounced temperature gradients appear. A cool-down phase starts when the pressure is recovered and is characterized by significant bath temperature gradients. The PI performance during the transient is poor, occasionally forcing the interlock system or the operator to adjust the control valves position in order to accelerate the cool-down or respect the refrigerator.
Fig. 5. LHC identification with CV 1.

Fig. 6. LHC identification with CV 2 at higher heat loads.

constraints on maximal vapor mass flow rates and its rate of change or to avoid overflowing of the phase separators.

Figures 5 and 6 present the circuit dynamics during two identification tests at different levels of static heat loads into the bath, corresponding to the LHC idle and approx. double of the LHC idle, respectively. The heat load can rapidly change as it depends on the qualities of the cryostat insulation vacuum and particles’ beam. During all tests electric heaters were used to imitate the additional heat loads. We observe that significant temperature gradients appear along the bath when the cooling power or the heat loads increase. The temperature dynamics changes drastically with increasing gradients: inverse response and dead-times are pronounced. The dynamics depends on amplitude and direction of the control valve movements and cell’s position.

III. THE MODEL

We briefly introduce the key elements of the detailed model that captures the key characteristics of the circuit dynamics in an LHC sub-sector at computational cost low enough to assure real-time feasibility of the NMPC and the MHSE. For a complete description see [6]. Its validity covers the full LHC operational range with He II. 1-D spatial discretization is done with $N_x = 28$ steps of variable length $\Delta x_i = 7.83$ m or 6.47 m, corresponding to half of the length of a cryo-dipole and the length of a Short Straight Section, respectively.

In an $i$-th finite volume, the bath temperature dynamics

$$dT_{h,i}(t)/dt = c_{v,i}^{-1} \Delta q_{h,i}(t)/\Delta M_{h,i}, \quad i = 1...N_x$$

(1)

with time $t$, identified helium mass $\Delta M_{h,i}$ and sum of heat transfers $\Delta q_{h,i}$. The inverse of He II specific heat capacity $c_{v,i}^{-1} \approx 7.99 \times 10^{-3} T_h^{-5.79} + 0.12 \times 10^{-3} (2.15 - T_h)^{0.32}$.

$$\Delta q_{h,i}(t) = \Delta q_{i}(t) - \Delta q_{c,i}(t) + q_{b,i-1}(t) - q_{h,i}(t)$$

(2)

involves the unknown sum of all heat loads $\Delta q_{i}$, including the electric heaters power $q_{EH,h}$, and cooling power $\Delta q_{c,i}$.

At a $j$-th interface between an $j$-th and $j + 1$-th finite volume, the expression for heat transfer in He II

$$q_{h,j} = \begin{cases} 
-(dT_h/dx)_{j}b_{\min,j}^{-1} & \text{if } b_{j}^{-1} < b_{\min,j}^{-1} \\
-(dT_h/dx)_{j} & \text{otherwise,}
\end{cases}$$

(3)

$$b_{j}^{-1} = |(dT/dx)_{h,j}|^{1-1/3} / A_{h,j} / T_h^{1/3},$$

(4)

with $j = 1...N_x - 1$, the identified bath cross-section $A_{h,j}$ and parameter $b_{\min,j}^{-1}$ limits the maximal apparent heat conductivity and thus stiffness of the temperature dynamics. At the bath extremities: $q_{b,0} = q_{h,N_x} = 0$. The He II Thermal Conductivity Function $F_j = 6.43 \times 10^{14} \left(1 + (t_j - 0.882)^2 \sum_{n=1}^{9} a_n (t_j - 1)^n \right)$, with $t_j = T_{h,j}/T_{\lambda}$ and constant coefficients $a_n$ [8].

The cooling power $\Delta q_{c,i} = \Delta q_{c,i} + \Delta q_{c,v,i}$ with heat transfers from the bath into the He II and vapor

$$\Delta q_{c,i} = \begin{cases} 
(T_{h,i} - T_{v,b,i}) C_{K,i} P_{l,b,i} \Delta x_i & \text{if } T_{h,i} \geq T_{v,b,i} \\
0 & \text{otherwise,}
\end{cases}$$

(5)

$$\Delta q_{c,v,i} = (T_{h,i} - T_{v,b,i}) C_{b,v,i} P_{v,b,i} \Delta x_i,$$  

(6)

flowing inside the heat exchanger, with the Kapitza conductance $C_{K,i}$, the vapor temperature $T_{v,b,i}$, thermal conductivity for the turbulent vapor flow $C_{b,v,i}$. The helium saturation temperature along the heat exchanger $T_{v,b,i}$ is related to the value at its extremity, at distribution line, $T_{h,b}(L)$ and to the pressure drop along the exchanger. The exchanger tube’s inner perimeter wetted by the vapor and He II $P_{v,b,i}$ and

$$P_{l,b,i} = \begin{cases} 
0.308 A_{l,b,i}^{0.289} & \text{if } A_{l,b,i} \geq A_{l,b,min,i} \\
0.308 A_{l,b,i} A_{l,b,min,i}^{-0.711} & \text{otherwise,}
\end{cases}$$

(7)

respectively, depend on heat exchanger cross-section occupied by He II $A_{l,b,i}$. The parameter $A_{l,b,min,i}$ is adjusted dynamically to limit the fastest dynamic modes and thus the stiffness. The mass conservation for the He II flow in the negative x direction

$$(dA/dt)_{l,b,i} = ((W_{l,b,i} - W_{l,b,i+1})/\Delta x_i - (dW_e/dx)_{b,i})/\rho_i$$

(8)

with He II mass flow rates at finite volume’s boundary

$$W_{l,b,j} = 6.25 \rho_i^{8/7} g^{4/7} \mu_i^{-1/7} (d\rho_d/dx)_{t} \mu_i^{4/7} A_{l,b,j}^{47},$$

(9)
with gravitational acceleration \( g \), He II density \( \rho_l \) and viscosity \( \mu_l \). He II evaporation rate in the exchanger

\[
(dW_e/dx)_{b,i} = (q_{c,i,t} + q_{i,f_2b,i})/\Delta h_{he}/\Delta x_i, \quad (10)
\]

The tunnel slope \((dy/dx)_{t} \) varies along the LHC. The heat exchanger He II overflow mass flow rates, at its extremities \( W_{i,b,0} \) and \( W_{l,b,N_s/2} \), affect the helium levels in the phase separator \( l_{ps} \).

The heat transfer from the feeding pipe to the He II

\[
q_{i,f_2b,i} = (T_{f,i} - T_{s,b,i}) C_{K_s}/2 \bar{P}_{f,i} \Delta x_i, \quad (11)
\]

with the temperature of the two-phase He I/He II-vapor flow in the pipe

\[
T_{f,i} = \begin{cases} 
T_{s,f,i,0} + (h_{f,i} - h_{f,i,0}) (dT_s/\Delta h_{s,f,i})_{f,i}, & \text{if } \chi_{f,i} = 0 \\
-4.43 + 2.57 \times 10^{-4} h_{f,i}, & \text{if } \chi_{f,i} = 1 \\
T_{s,f,i}, & \text{otherwise,}
\end{cases} \quad (12)
\]

depending on the value of the vapor quality \( \chi_{f,i} = (h_{f,i} - h_{s,f,i,0})/\Delta h_{he} \), with He II specific heat of evaporation \( \Delta h_{he} \). Helium enthalpy along the pipe \( h_{f,i} \) depends on enthalpy at the valves \( h_c \) and heat transfers. Heat is also transferred from the pipe to helium vapor in the exchanger affecting its enthalpy dynamics \( h_{v,b} \). \( h_{s,f,i} \) is the saturated helium enthalpy. Helium saturation temperature profile in the feeding pipe \( T_{s,f,i} \) depends on the pressure inside the exchanger and the pressure drop in the pipe. The mass of helium stored in the feeding pipe varies in function of the vapor quality, affecting the two-phase helium density \( \rho_{v,f,i} \), and thus the mass flow rate at its outlet, equal to that entering the heat exchanger, differs from that at the control valves \( W_e \).

Thus the model dynamics in the state space representation

\[
X_m(t + \Delta t) = f(X_m(t), U_m(t), P_m), \quad (13)
\]

\[
Y_m(t + \Delta t) = g(X_m(t), U_m(t), P_m). \quad (14)
\]

with the input \( U_m \in \mathbb{R}^{18} \), state \( X_m \in \mathbb{R}^{226} \), parameter \( P_m \in \mathbb{R}^{1} \) and output \( Y_m \in \mathbb{R}^{20} \)

\[
U_m = [(dq/dx)_{h,i}, q_{EH,h}, W_c, h_c, T_{s,b,L}]. \quad (15)
\]

\[
X_m = [T_h, \rho_f h A_1 h, h_{v,b}, \Delta t, P_{th}, T_{s,f}, T_{s,b}, W_{c,b}]. \quad (16)
\]

\[
P_m = [(dY/dx)_{t}]. \quad (17)
\]

\[
Y_m = [W_{l,b,0}, W_{l,b,N_s/2}, l_{ps}, T_{m,TT}], \quad (18)
\]

with the helium bath temperatures interpolated at the thermometers positions \( T_{m,0,TT} \). The system dynamics is integrated using forward Euler method with a relatively small time step \( \Delta t = 0.5 \) s imposed by the fastest modes of the stiff system dynamics, related to highly nonlinear heat transfer in He II and other mass and heat transfers. Many internal, slowly changing variables are updated each 20 simulation steps corresponding to 10 s of the simulation time. Dynamics of \( T_{s,b,i} \) and \( T_{s,f,i} \) do not represent the real very fast pressure dynamics, but a slower dummy dynamics used to solve complex algebraic loops at low computational cost, using continuation approach. The helium vapor mass flow rate inside the heat exchanger \( W_{v,c,b,i} \in \mathbb{R}^{28} \) affecting the \( T_{s,b,i} \) is a tearing variable in another algebraic loop.

IV. THE CONTROLLER

A. The MHSE-LO formulation

The state estimator provides the best possible estimates of the current states Eq. (16) and inputs Eq. (15) of the system, needed in the NMPC, using: 1) the model simulation, 2) the knowledge of physical constraints

\[
0 \leq W_{c,v,i}, \quad T_{s,r} \leq T_{s,b}(L), \quad (19)
\]

with cell index \( i_c = 1 \ldots 3 \) and the measurements vector

\[
M = [T_{TT,h} l_{LT,p} x_{CV,c} T_{TT,c} p_{PT,c} p_{PT,r} q_{EH,h}], \quad (20)
\]

composed of the magnets temperatures \( T_{TT,h} \in \mathbb{R}^{16} \), levels in the phase separators \( l_{LT,p} \in \mathbb{R}^{2} \), control valve positions \( x_{CV,c} \in \mathbb{R}^{2} \), helium temperatures and pressures at the valve \( T_{TT,c} \in \mathbb{R}^{2} \) and \( p_{PT,c} \in \mathbb{R}^{2} \), respectively, and the very low pressure at the refrigerator \( p_{PT,r} \in \mathbb{R}^{1} \).

In order to obtain the estimates at very low computational cost, the unconventional hybrid MHSE-LO approach is used, see Fig. 7. While the vector of MHSE optimized variables is strongly reduced to those having pronounced but strongly non-linear impact on the measured variables

\[
X_i(t) = [\Delta W_e, T_{s,b}(L)], \quad X_i \in \mathbb{R}^{3} \quad (21)
\]

with \( \Delta W_e = W_e - W_{c,th}(x_{CV,c} T_{TT,c} p_{PT,c}) \) representing the error between the estimated and theoretic helium mass flow rates at the valve. Both \( \Delta W_e \) and \( T_{s,b}(L) \) are assumed to be constant over the MHSE estimation horizon length \( T_e = 30 \) min. All the other variables are estimated using LO approach integrated into the MHSE. The heat load closes the loop in the LO,

\[
(dq/dx)_{h,i} = -5 e_{\Delta T,SC2} - 100 e_{T,h}, \quad (22)
\]

with the errors between observed and simulated magnet temperatures \( e_{\Delta T,h,SC2} = \sum_{n=1}^{16} (w_n (T_{h,0,TT,n} - T_{TT,n}) - w_{n-4} (T_{h,0,TT,n-4} - T_{TT,n-4})) \) and \( e_{T,h} = \sum_{n=1}^{16} w_n (T_{h,0,TT,n} - T_{TT,n})/16 \). The values of the weights...
\[ w_n \in [0,1] \text{ depend on sensor position and estimated current quality of measurement. In the MHSE, the model is}\]
\[ \text{simulated in the closed loop over a past estimation horizon. In order to allow the simulated variables to evolve to}\]
\[ \text{the consequent MHSE updates, at } j-\text{th MHSE execution, the system state at the beginning of the estimation time horizon}\]
\[ X^j_m(t - T_e) = X^j_m(t - T_e + \Delta t_c), \]  (23)
\[ \text{where } \Delta t_e = 20 \text{ s is the MHSE update time interval.}\]

The MHSE scheme is based on repeatedly solving the optimization problem:
\[ \min J_c, \text{s.t. } (13) \& (14) \& (22) \& (23) \& (19) \]  (24)
\[ J_c = \int_{t-T_e}^{t} (L_{TT} + L_{LT} + L_{W_c} + L_{q_i}) \, dt' \]  (25)
\[ \text{with cost functions representing the optimization objectives: 1) minimizing the error between the measured and estimated magnet temperatures } L_{TT} = 2 \cdot 10^3 \sum_{i=1}^{N_{Tm}=16} w_i (T_{h,i} - T_{TT,i})^2 \text{ and 2) incorporating approximate knowledge of the estimated variables’ values: } L_{LT} = (T_{s,b}(L) - (T_{s,cs} + \Delta T_{s,QRL}))^2, \]
\[ \text{with the approximate saturation pressure drop over the distribution line } \Delta T_{s,QRL}, L_{dq/dx} = ((dq/dx) - 0.1495)^2, \]
\[ L_{W_c} = \sum_{i=1}^{N_{SC}=2} ((W_{c,i})/(W_{c,i} + 10^{-4}))^2 \text{ and } L_{LT} = \sum_{i=1}^{N_{SC}} L_{LT,i}, \text{ using the available measurements only in a valid range } L_{LT,i} = \begin{cases} (l_p - l_{LT})^2 & \text{if } L_{LT,i} > 13 \% \\ 0 & \text{otherwise;} \end{cases} \]
\[ \text{This formulation fits a wide range of the operational conditions, with an exception of the situation when most of the measured bath temperatures are at the saturation temperature, when the observability of the system is degraded.}\]

\[ \text{B. The NMPC formulation}\]

The controller provides the values of manipulated variables corresponding to the best closed loop dynamics, according to the control objectives and respecting the constraints. The NMPC optimizes trajectories of the manipulated variables \( W_c(t') \), \( t \leq t' \leq t+T_e \) over the future time horizon length \( T_e = 2 \) h. The two trajectories are parameterized \( W_{c,isc}(t') = \sum_{i=1}^{3} \sigma_i(t') W_{c,isc,i}(t) \) using 6 parameters \( W_{c,isc,i} \), 3 per trajectory, and a set of \( N_b = 3 \) basis functions \( \sigma_i(t) \). The parametrisation with basis functions is suitable for the process with stiff dynamics, where model integration steps are much shorter than the length of the prediction horizon. Moreover, the basis function have form approximating an optimal solution and allowing to respect constraints on maximal input change rate. Thus the optimized variable
\[ X_c(t) = [W^*_c], \quad X_c \in \mathbb{R}^6. \]  (26)

The optimization problem is solved each \( \Delta t_e = 20 \) s:
\[ \min J_c, \text{s.t. } W^*_c(t) \geq 0, (13) \& (14) \]  (27)
\[ J_c = \phi_c(t + T_e) + \int_{t}^{t+T_e} (L_{SP} + L_{\Delta T} + L_{\Delta W_c} + L_{dW/dt_e} + C_{T_{max}} + C_{dW/dt_e} + C_{W_c} + C_{W_o}) \, dt', \]  (28)
\[ \text{with the cost functions representing the the control objectives: 1) minimizing the deviation from the setpoint } L_{SP} = 10^6 (\max (T_h) - T_{sp})^2, 2) leveling the bath temperature over the cell length } L_{\Delta T} = 400 (\max (T_h) - \min (T_h))^2, \]
\[ 3) \text{equilibrating the control valve positions } L_{\Delta W_c} = 10^6 (W_{cv,1} - W_{cv,2})^2 \text{ and 4) penalizing the variation of mass flow rates through the valves } L_{dW/dt_e} = \sum_{i=1}^{N_{SC}} 0.16 \cdot 10^{16} (dW_{cv}/d\tau_t \cdot dt_e)^2. \]

All non-box constraints on optimized variables are also integrated into cost function using the function \( f_c(x) = x^2 \) if \( x > 0 \) or \( f_c(x) = 0 \) otherwise. These address: 1) the maximal magnet temperatures \( C_{T_{max}} = \sum_{i=1}^{N_{SC}} 10^6 f_c(T_{h,i} - 2.1), 2) \]
\[ \text{the maximal positive and negative change rate of mass flow rate } C_{dW/dt_e} = 10^{16} f_c((dW_{cv,1}/d\tau_t + dW_{cv,2}/d\tau_t) - dW/dt_{max}), 3) \]
\[ \text{the maximal mass flow rates } C_{W_{cv}} = 10^{16} f_c(W_{cv,1} + W_{cv,2} - 7.7 \cdot 10^{-3}) \text{ and 4) the heat exchanger zero-overflow condition } C_{W_{out}} = \sum_{i=1}^{N_{SC}} 10^6 W_{out,t,b,i}^2 \]
\[ \text{The cost at the end of the horizon, approximates a cost functions integral from the horizon end to the infinity } \phi_e = 120 \cdot 10^3 \frac{1}{\max (T_h(T_e) - T_{sp}(T_e))^3}. \]

\[ \text{C. The numerical solver}\]

A simple, single-step Quasi-Newton approach with a variant of line-search is used to solve at each control update the non-linear optimization problems with box constraints Eq. (24) and Eq. (27). Using \( X \in \mathbb{R}^N_X \) to represent \( X_c \) or \( X_r \) and \( J \in \mathbb{R}^1 \) for \( J_c \) or \( J_r \) in case of MHSE and NMPC, respectively, the optimized value is updated as
\[ X^{j+1} = X^j + \alpha^j \Delta X^j, \]  (29)
\[ \text{with the } k\text{-th component of the search direction, } k = 1..N, \]
\[ (\Delta X^j)_k = \begin{cases} \frac{-1}{[J^j_{XX}]_k,k} [J^j_k]_k, & \text{if } [J^j_{XX}]_k,k > |[J^j_k]_k|/|[\Delta X_{d,m}]_k| \\ -\text{sgn}([J^j_k]_k) [\Delta X_{d,m}]_k, & \text{otherwise,} \end{cases} \]  (30)
\[ \text{is limited by the maximal step length component } |(\Delta X^j)_d| \leq |[\Delta X_{d,m}]_k|. \]
\[ \text{This formulation also assures that negative diagonal components of the Hessian } J_{XX} = d^2J/dX^2 \text{ have no impact on the search direction. The functionals } J_c \text{ and } J_r \text{ corresponding to } J, \text{ Eq. (25) and Eq. (28), are obtained through the model simulation over the estimation, or prediction, horizon and evaluating and integrating the cost functions. We use approximated values of the Jacobian vector } J_X = dJ/dX \]
\[ [J^j_X]_k,k \approx (J(X^j + \Delta X_k) - J(X^j) - J(X^j + \Delta X_k))/2|\Delta X_k|^2, \]
\[ \text{and of the Hessian’s diagonal elements } [J^j_{XX}]_k,k \approx (J(X^j + \Delta X_k) - 2 J(X^j) + J(X^j + \Delta X_k))/|\Delta X_k|^2, \]
\[ \text{where vector } \Delta X_k \text{ has zeros at all but the } k\text{-th position } \Delta X_{d,k} = [\Delta X]_k. \]
\[ \text{In order to numerically smoothen the non-smooth model, components of the vector } \Delta X_k \text{ have been chosen by trial and error, as the biggest positive values that do not visibly distort the approximated derivatives. The line search factor } \alpha^j \in [1, 1/2, 1/4, 1/8, 1/16, 1/32] \text{ is chosen that minimizes } J(X^{j+1}). \]
\[ \text{Box constraints are implemented by limiting the optimized variable value.} \]
The prototype controller has been implemented at the LHC as a PVSS II SCADA panel, accessing the process variables and sending values of manipulated variables to the PLCs of the process. The SCADA uses the controller’s C code through a control extension. Figures 8 and 9 present the results of the MHSE and NMPC validation at the LHC, performed at different static heat load levels: 1) approx. 0.18 W/m corresponding to the nominal LHC operation and 2) significantly increased to approx. 0.46 W/m, respectively. The scenario is the same for both cases: the first 40 W, 20 min long heat pulse into the bath corresponds to ramping up of magnet current during particles acceleration to 7 TeV and then additional heat load of 20 W is maintained corresponding to operation with beam. The two observed magnet temperatures envelopes, marked by the minimal and maximal temperatures, are extremely different. At higher heat loads: 1) the temperature gradients are huge, pronouncing large dead-times and inverse response of open loop dynamics, see Fig. 6, and 2) the lowest magnets temperature is saturated at the heat exchanger temperature, promoting helium accumulation in the heat exchangers and in the phase separators, observed with PI control, see Fig. 4. The NMPC/MHSE closed loop dynamics of the maximal magnets temperature is almost identical at the two very different heat loads: it is 10 times faster than that of the currently used PI and all the constraints are respected. However, the temperature stabilizes slightly under the set-point.

**REFERENCES**